2.3: (Continued)

$$\frac{dy}{dx} + P(x)y = q(x) \qquad \mu(x) = e^{\int P(x)dx}$$

$$\mu(x)\frac{dy}{dx} + P(x)\mu(x)y = q(x)\mu(x)$$

$$e^{\int P(x)dx}\frac{dy}{dx} + P(x)e^{\int P(x)dx}y = q(x)e^{\int P(x)dx}$$

$$\frac{d}{dx}\Big[\underbrace{\mu(x)}_{e^{\int P(x)dx}}y\Big] \underset{\text{or}}{=} \frac{d}{dx}\Big[e^{\int P(x)dx}y\Big] = q(x)e^{\int P(x)dx}$$

$$e^{\int P(x)dx}y = \int q(x)e^{\int P(x)dx}dx$$

$$\boxed{y = \frac{\int q(x)e^{\int P(x)dx}dx}{e^{\int P(x)dx}}} = \frac{\int q(x)\mu(x)dx}{\mu(x)}$$

Ex) 2.3 #14 $\quad \frac{xy'}{x} + \frac{(1+x)y}{x} = \frac{e^{-x}\sin(2x)}{x}$

$$y' + \underbrace{\left(\frac{1+x}{x}\right)}_{P(x)}y = \underbrace{\frac{e^{-x}\sin(x)}{x}}_{q(x)}$$

$$\mu(x) = e^{\int\frac{1+x}{x}dx} = e^{\int 1+\frac{1}{x}dx} = e^{\ln x + x} = e^{\ln x}e^{x}$$

$$\mu(x) = xe^{x}$$

$$\underbrace{xe^{x}y' + xe^{x}\left(\frac{1+x}{x}\right)y}_{} = \frac{e^{-x}\sin(2x)}{x}\cdot xe^{x}$$

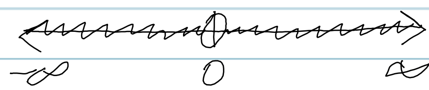
$$\frac{d}{dx}\left[xe^{x}y\right] = \sin(2x)$$

$$xe^{x} = \int \sin(2x)\,dx$$

$$xe^x y = -\frac{1}{2}\cos(2x) + C$$

$$\boxed{y = \frac{-\cos(2x)}{2xe^x} + \frac{C}{xe^x}}$$

Domain: $x \neq 0$



Interval: $\boxed{(0, \infty)}$

$\boxed{\text{transient terms}}$

$$\lim_{x \to \infty} e^{-x} = 0$$

Ex) 2.3 # 36 $\quad y' + \tan(x) y = \cos^2 x \quad y(0) = -1$

$$\mu(x) = e^{\int \tan(x)\,dx}$$
$$= e^{-\ln|\cos(x)|}$$
$$= e^{\ln|\cos(x)|^{-1}}$$
$$\boxed{= \frac{1}{\cos(x)}}$$

$$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx$$

$u = \cos(x)$
$du = -\sin(x)\,dx$

$$= -\ln|\cos(x)|$$

$$\frac{1}{\cos(x)} y' + \tan(x) \cdot \frac{1}{\cos(x)} y = \cos^2(x) \cdot \frac{1}{\cos(x)}$$

$$\frac{d}{dx}\left[\frac{1}{\cos(x)} y\right] = \cos(x)$$

$$\frac{1}{\cos(x)} y = \int \cos(x)\,dx$$

$$\frac{1}{\cos(x)} y = \sin(x) + C$$

$$\boxed{y = \sin(x)\cos(x) + C \cdot \cos(x)} \quad y(0) = -1$$

$$-1 = \sin(0)\cos(0) + C \cdot \cos(0)$$
$$-1 = C$$

$$\boxed{y = \sin(x)\cos(x) - \cos(x)}$$

Ex) 2.3 #38 $\frac{dy}{dx} + y = f(x)$ $y(0)=1$

$$f(x) = \begin{cases} 1 & 0 \le x < 1 \\ -1 & x > 1 \end{cases} \quad \begin{matrix} \text{part 1} \\ \text{part 2} \end{matrix}$$

Part 1 & 2

$P(x) = 1$ $\mu(x) = e^{\int P(x)dx} = e^{\int dx} = e^x$

Part 1

$$\underbrace{e^x \frac{dy}{dx} + e^x \cdot y}_{} = 1 \cdot e^x$$

$$\frac{d}{dx}\left[e^x y\right] = e^x$$

$$\int e^x y \, dx = \int e^x dx$$

$$e^x y = e^x + C$$

$$\boxed{y = 1 + Ce^{-x}}$$

Part 2

$$e^x \frac{dy}{dx} + e^x \cdot y = -1 \cdot e^x$$

$$\frac{d}{dx}\left[e^x y\right] = -e^x$$

$$e^x y = \int -e^x dx = -e^x + C$$

$$\boxed{y = -1 + Ce^{-x}}$$

$$y = \begin{cases} 1 + C_1 e^{-x} & 0 \le x < 1 \\ -1 + \cancel{C_2 e^{-x}}_{0} & x > 1 \end{cases}$$

$y(0) = 1$ Part 1 C_1

Part 2 $\lim_{x \to \infty} e^{-x} = 0$

$$1 = 1 + C_1 e^0$$

$$\boxed{0 = C}$$

$$\boxed{y = \begin{cases} 1 & 0 \le x < 1 \\ -1 & x > 1 \end{cases}}$$